THE DAUGAVET EQUATION FOR POLYNOMIALS ON C*-ALGEBRAS

Elisa Regina dos Santos

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DAUGAVET EQUATION

Daugavet Equation Appearance

- **I. K. Daugavet, 1963:** Every compact linear operator T on $C[0,1]$ satisfies the equation $\|\mathrm{Id} + T\| = 1 + \|T\|$.

Definition

Let X be a Banach space and let $T : X \to X$ be a bounded linear operator. We say that T satisfies the ***Daugavet equation*** *if*

$$\|\mathrm{Id} + T\| = 1 + \|T\|. \tag{DE}$$

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Examples

- **C. Foias and I. Singer, 1965:**
  Weakly compact linear operators on $C[0,1]$ satisfy (DE).
- **G. Y. Lozanovsky, 1966:**
  Compact linear operators on $L_1[0,1]$ satisfy (DE).
- **H. Kamowitz, 1984:**
  Compact linear operators on $C(K)$, where K is a compact Hausdorff space without isolated points, satisfy (DE).
- **J. R. Holub, 1987:**
  Weakly compact linear operators on $L_1(\mu)$, where μ is an atomless σ-finite measure, satisfy (DE).
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Alternative Daugavet Equation Appearance

- **J. Duncan, 1970:** If T is a bounded linear operator on $C(K)$, where K is a compact Hausdorff space, or if T is a bounded linear operator on $L_1(\mu)$, where μ is a σ-finite measure, then T satisfies the equation
$$\max_{|\lambda|=1} \|\mathrm{Id} + \lambda T\| = 1 + \|T\|.$$

Definition

*Let X be a Banach space and let $T : X \to X$ be a bounded linear operator. We say that T satisfies the **alternative Daugavet equation** if*

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Daugavet Equation for Polynomials

Generalization of (DE) and (ADE)

- X: Banach space
- B_X: closed unit ball of X
- X^*: topological dual of X

Definition

Let X be a Banach space and let Φ be a bounded mapping from B_X to X. We say that Φ satisfies the ***Daugavet equation*** *if*

$$\|\mathrm{Id} + \Phi\| = 1 + \|\Phi\|, \tag{DE}$$

and we say that Φ satisfies the ***alternative Daugavet equation*** *if*

$$\max_{|\lambda|=1} \|\mathrm{Id} + \lambda\Phi\| = 1 + \|\Phi\|. \tag{ADE}$$

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Examples

- **Y. Choi, D. García, M. Maestre and M. Martín, 2007:**
  If Ω is a completely regular Hausdorff space without isolated points, then every weakly compact polynomial on $C_b(\Omega, X)$ satisfies (DE).
- **Y. Choi, D. García, M. Maestre and M. Martín, 2008:**
  If μ is an atomless σ-finite measure, then every weakly compact polynomial on $L_\infty(\mu, X)$ satisfies (DE).
- **M. Martín, J. Merí and M. Popov, 2010:**
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The Polynomial Daugavet Equation on Commutative C*-algebras

Motivation

Theorem (Oikhberg (2002))

Let $\mathcal{A}$ be a C-algebra. Then every weakly compact linear operator on $\mathcal{A}$ satisfies* (DE) *if and only if $\mathcal{A}$ is non-atomic.*

Theorem (Martín & Oikhberg (2004))

Let $\mathcal{A}$ be a C-algebra. Then every weakly compact linear operator on $\mathcal{A}$ satisfies* (ADE) *if and only if all the atomic projections of $\mathcal{A}$ are central.*

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Some definitions

- $\mathcal{A}$: C*-algebra

Definition

*An element $p \in \mathcal{A}$ such that $p^2 = p = p^*$ is called a **projection**.*

Definition

*A non-zero projection $p \in \mathcal{A}$ is said to be **atomic** if, for every $a \in \mathcal{A}$, there exists $\lambda \in \mathbb{C}$ such that $pap = \lambda p$. $\mathcal{A}$ is said to be **non-atomic** if it has no atomic projections.*

Definition

*Let $\mathcal{A}$ be a commutative algebra. The **spectrum** of $\mathcal{A}$, denoted by $S(\mathcal{A})$, is the set of all non-zero homomorphisms from $\mathcal{A}$ into $\mathbb{C}$.*

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Representation of Commutative C*-algebras

Theorem

Let $\mathcal{A}$ be a commutative Banach algebra. Then $S(\mathcal{A})$ is a compact Hausdorff space for the Gelfand topology.

Gelfand-Naimark Theorem

Let $\mathcal{A}$ be a commutative C*-algebra. Then $\mathcal{A}$ is isometrically *-isomorphic to $C(S(\mathcal{A}))$.

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Definition

Let X be a complex Banach space. We denote by $\mathcal{A}_u(B_X, X)$ the Banach space of all uniformly continuous functions from B_X to X, which are holomorphic in the open unit ball.

Theorem (Choi, García, Maestre & Martín (2007))

Let S be a compact Hausdorff space without isolated points. Then every weakly compact $\Phi \in \mathcal{A}_u(B_{C(S)}, C(S))$ satisfies (DE).

Theorem 1

Let $\mathcal{A}$ be commutative C*-algebra. Then $\mathcal{A}$ is non-atomic if and only if $S(\mathcal{A})$ has no isolated points.

Results

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Theorem 2

Let $\mathcal{A}$ be a non-atomic commutative C*-algebra. Then every weakly compact Φ in $\mathcal{A}_u(B_{\mathcal{A}}, \mathcal{A})$ satisfies (DE).

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Let S be a compact Hausdorff space and let X be the complex space $C(S)$. Then every $\Phi \in \mathcal{A}_u(B_X, X)$ satisfies (ADE).

Proposition 3

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Some definition

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A complex Banach space U equipped with a continuous triple product

$$U \times U \times U \to U, \quad (a,b,c) \mapsto \{abc\}$$

is a ***JB*-triple*** *if it satisfies the following conditions.*

(1) *The triple product $(a,b,c) \mapsto \{abc\}$ is linear in a and c, and antilinear in b.*

(2) *The triple product is symmetric, that is, $\{abc\} = \{cba\}$.*

(3) *For every $x \in U$, the operator $D_x : U \to U$, defined by $D_x u = \{xxu\}$, is Hermitian (that is, $\|\exp(itD_x)\| = 1$ for all $t \in \mathbb{R}$) with non-negative spectrum.*

(4) *The "main identity"*

$$\{ab\{xyz\}\} = \{\{abx\}yz\} - \{x\{bay\}z\} + \{xy\{abz\}\}$$

is satisfied for all $a, b, x, y, z \in U$.

(5) *For every $x \in U$, $\|\{xxx\}\| = \|x\|^3$.*

Examples

- C*-algebras with the triple product

$$\{abc\} = \frac{ab^*c + cb^*a}{2}.$$

- Hilbert spaces with the triple product given in terms of the inner product $\langle \cdot, \cdot \rangle$ by

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Some definitions

Definition

Let x be an element of a JB-triple U. We define the antilinear operator $Q_x : U \to U$ by $Q_x(u) = \{xux\}$.*

Definition

An element $e \in U$ is called a ***tripotent*** *if $\{eee\} = e$.*

Definition

For any tripotent e, we define the ***Peirce projections****:*

$$P_2(e) = Q_e^2, \quad P_1(e) = 2(D_e - Q_e^2), \quad P_0(e) = \mathrm{Id} - 2D_e + Q_e^2.$$

Let $U_j(e)$ denote the range of $P_j(e)$ for $j = 0, 1, 2$. We say that

- *"e" is* ***minimal*** *if $U_2(e) = \mathbb{C}e$;*
- *"e" is* ***diagonalizing*** *if $U_1(e) = 0$.*

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Theorem

Let U be a JB-triple. Then every weakly compact operator on U satisfies* (DE) *if only if U has no minimal tripotents.*

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Definition

We denote by $\mathcal{P}_f(^mX, Y)$ the subspace of all $P \in \mathcal{P}(^mX, Y)$ which can be written in the form

$$P(x) = \sum_{i=1}^{k} y_i[x_i^*(x)]^m \text{ for any } x \in X,$$

where $y_i \in Y$ and $x_i^* \in X^*$. We denote by $\mathcal{P}_f(X, Y)$ the algebraic sum of spaces $\mathcal{P}_f(^mX, Y)$ with $m \in \mathbb{N}_0$. Each $P \in \mathcal{P}_f(X, Y)$ is said to be a ***polynomial of finite type****.*

Definition

We denote by $\mathcal{P}_A(^mX, Y)$ the closure of $\mathcal{P}_f(^mX, Y)$ in $\mathcal{P}(^mX, Y)$ and we denote by $\mathcal{P}_A(X, Y)$ the algebraic sum of spaces $\mathcal{P}_A(^mX, Y)$ with $m \in \mathbb{N}_0$. The elements of $\mathcal{P}_A(X, Y)$ are called ***approximable polynomials****.*

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Results about JB*-triples

Theorem 4

Let U be a JB*-triple. The following are equivalent:

1. U has no minimal tripotents;
2. Every polynomial of finite type $P : U \to U$ satisfies (DE);
3. Every approximable polynomial $P : U \to U$ satisfies (DE);
4. Every rank one bounded linear operator $T : U \to U$ satisfies (DE).

Results about JB*-triples

Theorem 5

Let U be a JB*-triple. The following are equivalent:

1. All minimal tripotents of U are diagonalizing;
2. Every polynomial of finite type $P : U \to U$ satisfies (ADE);
3. Every approximable polynomial $P : U \to U$ satisfies (ADE);
4. Every rank one bounded linear operator $T : U \to U$ satisfies (ADE).

Results about C*-algebras

Theorem 6

Let $\mathcal{A}$ be a C*-algebra. Then:

1. $\mathcal{A}$ is a JB*-triple;
2. $\mathcal{A}$ has atomic projections if and only if it has minimal tripotents;
3. All atomic projections of $\mathcal{A}$ are central if and only if all of its minimal tripotents are diagonalizing.

Results about C*-algebras

Theorem 7

Let $\mathcal{A}$ be a C*-algebra. The following are equivalent:

1. $\mathcal{A}$ is non-atomic;
2. Every polynomial of finite type $P : \mathcal{A} \to \mathcal{A}$ satisfies (DE);
3. Every approximable polynomial $P : \mathcal{A} \to \mathcal{A}$ satisfies (DE);
4. Every rank one bounded linear operator $T : \mathcal{A} \to \mathcal{A}$ satisfies (DE).

Results about C*-algebras

Theorem 8

Let $\mathcal{A}$ be a C*-algebra. The following are equivalent:

1. All atomic projections of $\mathcal{A}$ are central;
2. Every polynomial of finite type $P : \mathcal{A} \to \mathcal{A}$ satisfies (ADE);
3. Every approximable polynomial $P : \mathcal{A} \to \mathcal{A}$ satisfies (ADE);
4. Every rank one bounded linear operator $T : \mathcal{A} \to A$ satisfies (ADE).

Open problems

- If $\mathcal{A}$ is a non-atomic C*-algebra, then every weakly compact linear operator on $\mathcal{A}$ satisfies (DE). Is there a similar result for polynomials?
- If $\mathcal{A}$ is a C*-algebra whose atomic projections are central, then every weakly compact linear operator on $\mathcal{A}$ satisfies (ADE). Is there a similar result for polynomials?

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Main References

E. R. Santos. *A equação de Daugavet para polinômios em espaços de Banach*, Doctoral dissertation, IMECC-UNICAMP, 2013.

Y. S. Choi, D. García, M. Maestre, M. Martín. *The Daugavet equation for polynomials*. Studia Math. **178** (2007), 63-82.

I. K. Daugavet. *On a property of completely continuous operators in the space C*. Uspekhi Mat. Nauk **18** (1963), 157-158 (em russo).

M. Martín, T. Oikhberg. *An alternative Daugavet property*. J. Math. Anal. Appl. **294** (2004), 158-180.