# Symposium in Real Analysis XXXVII



# **Book of Abstracts**



**The Sugarcane Symposium** 

#### Welcome

The Instituto de Ciências Matemáticas e de Computação of the Universidade de São Paulo (ICMC-USP), the Instituto Nacional de Ciência e Tecnologia de Matemática (INCTMat) and the Real Analysis Exchange welcome you to the Symposium in Real Analysis XXXVII which will take place at the city of São Carlos, in the heart of the state of São Paulo, Brazil, on June 3-6, 2013. We kindly call this symposium THE SUGARCANE SYMPOSIUM after the plantations of sugarcane nearby.

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### Address

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Maps

#### Entrances an Exits of the Campus

- 1. ICMC entrance
- 2. Main entrance
- 3. Observatório's entrance
- 4. Physics institute's entrance
- 5. Physics institute's exit

#### Hotels

- 1. Hotel Indaiá Residence
- 2. Parisi Hotel
- 3. Hotel Indaiá
- 4. Hotel Anacã
- 5. San Ciro Apart Hotel
- 6. Atlantic Inn Residence

#### Bars

- 1. Tio Joaquim
- 2. Boteco Santa Teresa
- 3. Vila Brasil Botequim
- 4. Donna Léo Choperia
- 5. Mosaico Bar e Restaurante
- 6. Cachaçaria Água Doce
- 7. Almanach Café e Restaurante
- 8. Pimentas Bar
- 9. Seo Gera

#### Restaurants

- 1. Restaurante La Salute (lunch only)
- 2. Restaurante La Villa
- 3. Casa do Café
- 4. Cantina Ciao Bello
- 5. Mosaico Bar e Restaurante
- 6. Sabor Oriental

- 7. Restaurante Mamãe Natureza (lunch only)
- 8. Restaurante Barone
- 9. Kalil Cozinha Árabe
- 10. Roda Chopp
- 11. Sushi-Ya San
- 12. Restaurante Panela
- 13. Restaurante Curinga

#### Pizzerias (dinner only)

- 1. Pizzaria Bom Pedaço
- 2. Pizzaria Amici
- 3. Pizzaria Don Raffaele

#### Money Exchange

1. Fitta Câmbio







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General Information

#### Conference site

The meeting will take place at Buildings 6 and also in the Coffee Area. See the map on page 11.

#### Registration

The registrations will be made in the following schedule:

- Sunday, June  $2^{nd}$  From 17:00hs to 18:00hs in the lobby of Anacã Hotel.
- Monday, June  $3^{rd}$  From 8:00hs to 8:20hs in the entrance of the Auditorium (Building 6).

We will provide you a badge at registration. Please wear your badge at the event.

The Event's Office (number 3, map on page 11) will be at your disposal for any questions and information.

#### Financial support

The financial support payment is planned to be made on Tuesday, June  $4^{th}$  in the Financial Office (number 4, map on page 11).

#### Meals and refreshments

There are several restaurants near the campus. You can find them by looking at the city map on page 5 (the marks in red). There are also three choices of pizzerias, which are marked in brown. At night, there are many bars around the city marked in blue.

There is a canteen available at ICMC where you can have either snacks or lunch. Look for Building 7 on the map on page 11.

#### Social events

- Monday, June  $3^{rd}$ : Cocktail at 18:00 at the Coffee Area.
- Tuesday, June  $4^{th}$ : Photo of the meeting at 12:10 at ICMC.
- *Wednesday, June* 5<sup>th</sup>: Barbecue from 12:00 to 18:00 at Sítio do Moinho. Transportation from ICMC to Sítio do Moinho and back will be provided.

#### Health emergencies

In case of accidents or health emergencies call 192 (SAMU).

#### Money exchanges

In case you need to exchange your money, we recommend you to look for Fitta Câmbio, located at Rua Episcopal, 1931. The working hour is from 9:00 to 17:30. You can locate it at the map in page 7.

#### Smoking

Smoking is prohibited in any of the ICMC buildings.

#### Computer and wireless LAN use

There will be available computers and a printer for use at Event's Office (look for Event's Office (3) at the ICMC map located in page 11).

In order to access the wireless connection at the University you need to follow the steps:

- 1. Enable wireless on your device.
- 2. Join the ICMC-GUEST wireless network.
- 3. Open a browser and attempt to visit a website (for example your home page).
- 4. Click on the button in the page to proceed.
- 5. You will be redirected to a login page. Enter the login and password as follows:

login: *eventos* password: icmc!2013

6. You may freely browse the internet after logging in. You may occasionaly need to re-authenticate using the above procedure.

Plenary talks

#### Thermodynamic formalism for one-dimensional spin-lattices: Entropy and pressure

#### Artur Lopes

Universidade Federal do Rio Grande do Sul, Brazil

We generalize several results of the classical theory of Thermodynamic Formalism when the state space is a general compact metric space M. We consider the shift acting on  $M^{\mathbb{N}}$  and a general a priori measure for defining the Ruelle operator.

We define entropy and by its very nature it is always a nonpositive number. If M is not finite there exists Gibbs states with arbitrary small negative value. Given a Lipchitz interaction potential defined on the lattice, we consider the Pressure problem and its relation with eigenfunctions and eigenprobabilities of the Ruelle operator. The concepts of entropy and Ruelle operator are linked.

The so called XY model fits under our setting. In this case M is the unitary circle  $S^1$ . We explore the differentiable structure of  $(S^1)^{\mathbb{N}}$  considering interaction potentials which are of class  $C^2$  and the corresponding eigenfunctions.

This is a joint work with J. Mengue, J. Mohr and R. Souza.

#### Linear subsets of nonlinear sets in topological vector spaces Daniel Marinho Pellegrino

Universidade Federal da Paraíba, Brazil

For the last decade there has been a generalized trend in Mathematics on the search for algebraic structures (linear spaces, closed subspaces, or infinitely generated algebras) composed of mathematical objects enjoying certain properties. This trend has caught the eye of many researchers and has also had a remarkable influence in Real and Complex Analysis, Operator Theory, Summability Theory, Polynomials in Banach spaces, Hypercyclicity and Chaos, and general Functional Analysis. We try to provide an account on the advances and on the state of the art of this trend, nowadays known as lineability and spaceability. This is a joint work with Luis Bernal and Juan Seoane.

#### Universality in the Laguerre–Pólya class

#### Dimitar Dimitrov

Universidade Estadual Paulista Júlio de Mesquita Filho, Brazil

The Laguerre-Pólya class, denoted by  $\mathcal{LP}$ , consists of entire functions which are limits, in the sense of local uniform convergence, of polynomials with only real zeros. Interesting and important functions from  $\mathcal{LP}$  are those represented as Fourier transforms of certain positive Borel measures. The problem to characterize the latter subclass was posed by George Pólya in 1926 who was motivated by his efforts to settle the Riemann hypothesis. Essentially the same question arose in Statistical Mechanics and it is related to the celebrated Lee-Yang circle theorem, for which Lee and Yang were awarded the 1957 Nobel Prize in physics. We report a result which provides a characterization of the so-called Lee-Yang measures and a solution of Pólya's problem too, in terms of the polynomials, orthogonal with respect to the measure.

We shall discuss also another conjecture of Pólya which states that the real line attracts the zeros of the entire functions of order less than two under differentiation. Though the conjecture was settled by Craven, Csordas and Smith, further results indicate that once such a function is differentiated, the zeros not only approach the real line but also become equally spaced.

#### Extremal problems and Hilbert spaces of entire functions

#### **Emanuel Carneiro**

Instituto de Matemática Pura e Aplicada, Brazil

In this talk we will present a class of extremal problems connected to the theory of de Branges spaces of entire functions, in which one wants to approximate a given function by entire functions of exponential type minimizing the error in the de Branges metric. A fundamental tool to solve such a problem is to construct entire interpolations at the zeros of Pólya-Laguerre functions. Ultimately, we shall specialize the construction to a certain family of homogeneous de Branges spaces, in order to establish a connection to extremal problems in Euclidean spaces and applications to inequalities in number theory.

#### Non-absolutely convergent integrals with respect to distributions Jan Malý

Charles University in Prague, Czech Republic

We introduce integrals of functions with respect to distributions (in the Euclidean setting) or distribution-like functionals on metric spaces. The integral is nonabsolutely convergent, similarly to the Denjoy-Perron (or Henstock-Kurzweil) integral. The process of integration is new even for integration with respect to the Lebesgue measure on the real line. Applications in spirit of Stokes' theorem are indicated. This is a joint work with Petr Honzík and Kristýna Kuncová.

#### The Kurzweil-Henstock integral and its extensions : a historical survey

#### Jean Mawhin

Université Catholique de Louvain, Belgium

We survey the evolution of the Kurzweil-Henstock integral and of its main extensions to functions of several variables.

#### On functions with continuous restrictions to various sets Krzysztof Chris Ciesielski

West Virginia University, USA

A function  $f: \mathbb{R}^n \to \mathbb{R}$  is separately continuous, when it has continuous restriction  $f \upharpoonright H$  for all hyperplanes H perpendicular to one of the axes. We will briefly discuss a classical work on this class of functions and explore different generalizations of this notion. In one direction, we will examine maps  $f: \mathbb{R}^n \to \mathbb{R}$  with continuous restrictions to: (a) all k-dimensional flats (i.e., affine subspaces) of  $\mathbb{R}^n$ ; (b) only the flats that are parallel to subspaces generated by coordinate axis. For each of these classes, we examine the structure of the sets of points discontinuities of its members and show that these families of sets are different for different classes. In another direction, we investigate for what classes Gof functions  $g: \mathbb{R} \to \mathbb{R}$  the continuity of  $f: \mathbb{R}^2 \to \mathbb{R}$  is insured by the continuity of all its restrictions  $f \upharpoonright g$  for  $g \in G$ . We note, that this is the case for G consisting of all continuously differentiable functions, if we allow infinite derivatives; however, the result is false for twice differentiable functions. We also show, that the continuity of f can be insured for G consisting of all translations of a single Baire class one function g; however, this cannot be achieved when g is continuous.

#### Generalized linear differential equations in a Banach space: Continuous dependence on a parameter

#### Milan Tvrdý

Academy of Sciences of the Czech Republic, Czech Republic

The contribution is based on the joint research with Giselle A. Monteiro. The aim is to present new conditions ensuring the continuous dependence on a parameter k of solutions to linear integral equations of the form

$$x(t) = \tilde{x}_k + \int_a^t d[A_k] x + f_k(t) - f_k(a), \quad t \in [a, b], k \in \mathbb{N},$$

where  $-\infty < a < b < \infty$ , X is a Banach space, L(X) is the Banach space of linear bounded operators on X,  $\tilde{x}_k \in X$ ,  $A_k : [a, b] \to L(X)$  have bounded variations on [a, b],  $f_k : [a, b] \to X$  are regulated on [a, b]. The integrals are understood as the abstract Kurzweil-Stieltjes integrals and the studied equations are usually called generalized linear differential equations (in the sense of J. Kurzweil).

Our main theorem concerns the case when the variations  $var_a^b A_k$  need not be uniformly bounded and it is an analogy of the Opial's result (JDE, 1967) for ODEs.

Applications to linear dynamic equations on time scales are then enabled by their relationship with generalized differential equations as disclosed by A. Slavík (JMAA, 2012).

#### Non-integrable functions

#### Patrick Muldowney

#### University of Ulster, Northern Ireland

From ancient problems of quadrature down to modern problems of communications, fundamental physics, and financial asset pricing, mathematical analysis has developed by formulating and resolving subtle and challenging issues of integration. A historic thread of ideas, including the method of exhaustion, the method of indivisibles, the law of tangency (or fundamental theorem of calculus), the theory of measure, and the method of Cauchy sums and Riemann sums, connects the work of successive analysts.

#### Differentiation properties related to the Keleti perimeter to area conjecture

#### Paul D. Humke

St. Olaf College and Washington and Lee University, USA

We examine finite unions of unit squares in same plane and consider the ratio of perimeter to area of these unions. In 1998, T. Keleti published the conjecture that this ratio never exceeds 4. Here we study the continuity and differentiability of functions derived from the geometry of the union of those squares. Specifically we show that if there is a counterexample to Keleti's conjecture, there is also one where the associated ratio function is differentiable.

#### After the Bishop-Phelps Theorem

#### **Richard Martin Aron**

Kent State University, USA

A mapping between normed spaces,  $F : X \to Y$ , is said to be *norm-attaining* if there is  $x_0 \in X$ ,  $||x_0|| = 1$ , such that  $\sup_{x \in B_X} ||F(x)|| = ||F(x_0)||$ . (Here,  $B_X$  is the closed unit ball of X.) This *expository* talk has its origins in work by Victor Klee in the 1950's, but our discussion will begin with the following short and elegant 1961 result of Errett Bishop and Robert Phelps.

**Theorem:** Let  $\varepsilon > 0$ . For any X and any continuous linear form  $\varphi \in X^*$ , there is a norm-attaining element  $\psi \in X^*$  such that  $\|\varphi - \psi\| < \varepsilon$ .

Our intention is to describe some of the branches of study that are direct results of this 1-1/2 page paper during the last 50+ years. Among the topics that we plan to discuss are norm-attaining linear operators and norm-attaining multilinear functions.

Contributed talks

#### Petri nets through cone groups

#### Antonio Roberto da Silva

Universidade Federal do Rio de Janeiro, Brazil

In a landmark paper by C.A. Petri (Internat. J. Theoret. Phys., 1982), it was established the foundations of the important connection between physical and computational processes involving discrete structures. The resulting theory has known an enormous growth with multifarious applications. On the other hand, from the physical side, R. D. Sorkin provided an interesting approach to continuous topology as a sort of limit of finitary ones (see the paper by R. D. Sorkin in Internat. J. Theoret. Phys., 1991). Furthermore, S. P. Novikov and M. Requardt have implemented this finitary viewpoint for distinguished specific operators (see the papers by S.P. Novikov in Fields Institute Comm., 1999 and by M. Requardt in J.Phys.A, 2002). In this presentation we sketch the foundational aspects of a group theoretical approach to Petri nets as presented in the papers by E. Badouel, J. Chenou and G. Guillou in Lecture Notes in Computer Science, v.3580, 2005, and by B. Brosowski and A. R. da Silva in Workshop Algorithmen und Werkzeuge für Petrinetze, 1998. This algebraic viewpoint makes possible to treat different types of Petri nets in a unified way and raises hope to a possible extension to certain C\*-algebras, see e.g. the paper by D. Mundici in J. Funct. Anal., 1986.

### Nonlinear operators in the space of functions of bounded variation in the sense of Jordan

#### Dariusz Bugajewski

Adam Mickiewicz University, Poland

It is well-known that solutions to many differential and integral equations which describe concrete physical phenomena are functions of bounded variation in the sense of Jordan. From that perspective the Banach space of such functions forms an interesting place to examine classical nonlinear operators such as convolution operators, nonlinear superposition operators as well as nonlinear integral operators. The aim of this talk is to present mainly a collection of new results in this area. Particular attention will be paid to nonautonomous superstition operators since several interesting problems in this area have been open for many years.

#### Lyapunov theorems for measure FDEs via Kurzweil-equations

#### Eduard Toon

Universidade Federal de Juiz de Fora, Brazil

This is a joint work with Márcia Federson and Jaqueline G. Mesquita. We consider measure functional differential equations (we write measure FDEs) of the form  $Dx = f(x_t, t)Dg$ , where f is Perron-Stieltjes integrable,  $x_t$  is given by  $x_t(\theta) = x(t + \theta), \theta \in [-r, 0]$ , with r > 0, and Dx and Dg are the distributional derivatives in the sense of the distribution of L. Schwartz, with respect to functions  $x : [t_0, \infty) \to \mathbb{R}^n$  and  $g : [t_0, \infty) \to \mathbb{R}$ ,  $t_0 \in \mathbb{R}$ . We establish a Lyapunov-type theorem for measure FDEs via theory of generalized ordinary differential equations (also known as Kurzweil equations).

### An application of Wazewski's principle to an existence problem in celestial mechanics

Anete S. Cavalcanti and Eduardo Shirlippe Goes Leandro

Universidade Federal de Pernambuco, Brazil

In 1979, R. Broucke published a numerical proof of existence for a symmetrical periodic solution of the planar isosceles three-body problem. Broucke's solution turned out to be very important in Astronomy as an example of stable periodic orbit in the equal-mass Newtonian three-body problem (only two other such orbits are known). An analytical proof, which uses the calculus of variations, was given by Shibayama in 2011. We provide a constructive, topological proof of existence of Broucke solutions with a strong dynamical systems flavor.

#### The Daugavet equation for polynomials on C\*-algebras

#### Elisa Regina dos Santos

Pontifícia Universidade Católica - Campinas, Brazil

The Daugavet equation and the alternative Daugavet equation were originally defined for linear operators in the works of I. K. Daugavet and of M. Martín and T. Oikhberg. Recently Y. S. Choi et al generalized their definitions for bounded functions on Banach spaces and these have been studied in particular for polynomials and homogeneous polynomials. We investigate such equations for polynomials on  $C^*$ -algebras. We study separately the cases in which the  $C^*$ -algebras are commutative and non-commutative. In these cases, we obtain necessary and sufficient conditions on such  $C^*$ -algebras in order that certain polynomial classes satisfy the Daugavet equation and the alternative Daugavet equation.

#### Some results related with the Riemann-Lebesgue Lemma II Francisco Javier Mendoza Torres

Benemérita Universidad Autónoma de Puebla, Mexico

We obtain a generalization of the Riemann-Lebesgue lemma for functions in  $BV_0(\mathbb{R})$ , the space of bounded variation functions that vanish at infinity. It is known that, in general, this lemma is not valid for not Lebesgue integrable functions. The result is of interest because of  $BV_0(\mathbb{R})$  do not have inclusion relations with the Lebesgue space. Moreover we get a class of functions in  $BV_0(\mathbb{R})$ which are Henstock-Kurzweil integrable but not Lebesgue integrable. As a consequence, we prove the Dirichlet-Jordan theorem in  $BV_0(\mathbb{R})$ .

#### Stationary phase and quantum momentum maps Igor Mencattini

Universidade de São Paulo, Brazil

In this talk I will explain how to get an explicit formula for a family of deformation quantizations for the momentum map associated with the cotangent lift of a Lie group action on a finite dimensional vector space (this is a joint work with Benoit Dherin).

#### Volterra integral equations on time scales

Iguer Luis Domini dos Santos

Universidade Estadual Paulista Júlio de Mesquita Filho, Brazil

We consider a class of Volterra integral equations on time scales already established in the literature and we extended for this class a convergence result of solutions. As an application of this result, we obtain the existence of solutions to optimal control problems in time scales described by this class of integral equations on time scales.

#### Method of intersection of spheres used to calculation of protein structures

Inajara da Silva Freitas and Luiz Carlos Matioli

Universidade Federal do Paraná, Brazil

In this work, based on a paper by Dong, Qunfeng and Wu, Zhijun, which appeared in J. Global Optim. in 2002, we discuss the problem of determining the structure of a protein when a set of distances between the atoms are known. This problem is also known as molecular geometrical problem away, we will assume that all distances between the atoms are known, which will be addressed two different views. First, we formulate the problem so that it can be solved linearly, using basic techniques of linear algebra. Then we'll talk about the quadratic method in which the problem is formulated using the calculation of the intersection points between spheres. To find the points intersection between spheres in  $\mathbb{R}^n$ , we will suppose to have a set of n nonlinear equations, where we want to find it's solution by solving a system of quadratic equations. The methods were implemented and tested in MATLAB and then compared using the Root-Mean-Square-Deviation (RMSD), which measures the accumulated error in calculating these structures.

#### Almost automorphic solutions of dynamic equations on time scales

Carlos Lizama and Jaqueline Godoy Mesquita

Universidade de São Paulo, Brazil

The theory of time scales is a recent theory which started to be developed by Stefan Hilger, on his doctoral thesis in order to unify the discrete and continuous cases. However, since time scale is a closed and nonempty subset of  $\mathbb{R}$ , this theory can unify several cases, depending on the chosen time scale.

In the present work, we introduce the concept of almost automorphic functions on time scales and present results about their properties. Then, we study the nonautonomous dynamic equations on time scales given by  $x^{\Delta}(t) = A(t)x(t) + f(t)$  and  $x^{\Delta}(t) = A(t)x(t) + g(t, x(t))$ ,  $t \in \mathbb{T}$ , where  $\mathbb{T}$  is an invariant under translations time scale,  $A \in \mathcal{R}(\mathbb{T}, \mathbb{R}^{n \times n})$ ,  $f \in C_{rd}(\mathbb{T}, \mathbb{R}^n)$  and  $g \in C_{rd}(\mathbb{T} \times \mathbb{R}^n, \mathbb{R}^n)$ . We prove a result ensuring the existence of an almost automorphic solution for both equations, assuming that the associated homogeneous equation of this system admits an exponential dichotomy, A(t) is almost automorphic and nonsingular matrix function,  $A^{-1}(t)$  and  $(I+\mu(t)A(t))^{-1}$  are bounded for every  $t \in \mathbb{T}$ , f is almost automorphic function and g is almost automorphic function with respect to first variable. Also, assuming the function g satisfies the global Lipschitz type condition, we prove the existence and uniqueness of an almost automorphic solution of the nonlinear dynamic equation on time scales. Further, we present some applications of our results for some new almost automorphic time scales. Finally, we present some interesting models which our main results can be applied.

#### Metrizability of Lévy topology on nonadditive measures

#### Jun Kawabe

Shinshu University, Japan

Weak convergence of measures on a topological space not only plays a very important role in probability theory and statistics, but is also interesting from a topological measure theoretic view, since it gives a convergence closely related to the topology of the space on which the measures are defined. Thus, it is possible to study weak convergence of measures on a topological space in association with some topological properties of the space, such as the metrizability, separability and compactness. Nonadditive measures, which are set functions that are monotonic and vanish at the empty set, have been extensively studied; see Wang and Klir. They are closely related to nonadditive probability theory and the theory of capacities and random capacities. Nonadditive measures have been used in expected utility theory, game theory, and some economic topics under Knightian uncertainty. The notion of weak convergence of nonadditive measures was formulated by Girotto and Holzer in a fairly abstract setting. Some of their fundamental results for weak convergence, such as the portmanteau theorem and the direct and converse Prokhorov theorems, have been extended to the nonadditive case. In particular, the portmanteau theorem allows us to show that the weak topology, which is the topology generated by weak convergence, coincides with the Lévy topology, which is the topology generated by convergence of measures on a special class of sets. In this talk, we will present successful nonadditive analogs of the theory of weak convergence of measures with a particular focus on metrizability and we will also supply weak convergence methods to related fields; see the authors recent paper. This work is supported by Grant-in-Aid for Scientific Research No. 20540163, Japan Society for the Promotion of Science (JSPS).

#### Removable singularities for the equation $\operatorname{div}(v) = 0$

Laurent Moonens

Universite Paris-Sud 11 (Orsay), France

In this talk, we shall present some new results in the study of removable singularities for the equation  $\operatorname{div}(v) = 0$ . Recall that a (compact) set is called removable for the above equation with respect to a class B of measurable vector fields in  $\mathbb{R}^n$  if for any  $v \in B$ , the condition  $\operatorname{div}(v) = 0$  outside S implies  $\operatorname{div}(v) = 0$  in  $\mathbb{R}^n$ . We shall in particular explain how characterizing removable sets relies heavily on integrability and/or continuity properties of the elements of B.

### Regulated solutions and periodicity for nonlinear integral equations on time scales in Banach spaces

Luciano Barbanti, Berenice C. Damasceno and Camila A. Martins

Universidade Estadual Paulista Júlio de Mesquita Filho, Brazil

This work deals with a large class of controlled nonlinear integral equations on time scales (see S. Hilger, M. Bohner) in Banach spaces where the integral used is the right Cauchy-Stieltjes one(see L. Barbanti, M. Federson, B.C. Damasceno, G.N. Silva). By considering the concepts of -contraction and periodic points (see M. Edelstein, R. Demarr, S.B. Nadiel, R.E. Smithson) we give conditions on the kernel in the integral for the existence of solutions, periodic solutions and uniqueness in the nonlinear equation.

### Perturbation of dynamical systems defined by semilinear parabolic problems

#### Marcone Correa Pereira

Universidade de São Paulo, Brazil

In this talk we analyze the dynamics of flows generated by a class of dissipative semilinear parabolic problems when some parameters of the equation vary in a topological space. We establish abstract results and apply them to partial differential equations with nonlinear boundary conditions when (i) the domain of definition of the solutions vary with respect to the action of diffeomorphisms, and (ii) when some reaction and potential terms of the equation are concentrating in a narrow strip of a portion of the boundary of the domain of the solutions. Our main goal is to discuss the continuity of the nonlinear semigroup, as well as, the upper and lower semicontinuity of the family of attractors.

### Generalized second order Cauchy problems of differential inclusion with nonlocal and impulsive condition

#### Marcos Napoleão Rabelo

Universidade Federal de São Paulo, Brazil

In this work, we study the existence of second order cauchy problem associated with a model involving nonlocal and impulsive conditions. We deal with differential inclusions, and to achieve our results we use the so called measure of non-compactness.

#### Some results related with the Riemann-Lebesgue Lemma I

#### María Guadalupe Morales Macías

Benemérita Universidad Autónoma de Puebla, Mexico

We obtain some versions of the Riemann-Lebesgue Lemma in the Henstock-Kurzweil (HK) Integral context. In general, this result is not valid on HK[a, b]. On the HK space completion (HK([a, b])) the following asymptotic behavior holds.

Let [a, b] a compact interval. If  $\varphi$  is a function on [a, b] to  $\mathbb{R}$  such that  $\varphi'$  is bounded and  $\varphi(s) = o(s)$ when  $|s| \to \infty$ , then for each  $f \in H\widehat{K([a, b])}$ 

$$\int_{a}^{b} \varphi(st) f(t) dt = o(s), \ |s| \to \infty.$$
(1)

Because of the functions sine and cosine are such that sin(s) = cos(s) = o(s),  $s \to \infty$ , then the Fourier Coefficients have a similar behavior as the expression (1). So with this result we obtained Fourier Series partial sum has an asymptotic behavior as the expression (1), for elements that are in the completion.

The Riemann-Lebesgue Lemma is related to the convergence of integrals of products of the form  $fg_{\lambda}$ , where f and  $g_{\lambda}$  satisfy certain conditions for the product to be integrable in some sense. In this case, if  $f \in H\widehat{K([a, b])}$  and g is a bounded variation function on  $\mathbb{R}$ , then

$$\lim_{\lambda \to \infty} \int_{-\infty}^{\infty} f(t)g(\lambda t)dt = \lim_{t \to \infty} g_{+}(t) \int_{0}^{\infty} f(t) + \lim_{t \to \infty} g_{-}(t) \int_{-\infty}^{0} f(t) dt = \lim_{t \to \infty} g_{+}(t) \int_{0}^{\infty} f(t) dt = \lim_{t \to \infty} g_{+}(t) \int_$$

We can note that  $\lim_{t\to\infty} g_+(t)$  and  $\lim_{t\to\infty} g_-(t)$  always exist.

If we consider  $A_{\lambda}=\int_{a}^{b}f(t)g(\lambda t)$  as new coefficients, the serie generated is

$$\dot{S}(f,t) = \sum_{\lambda=0}^{\infty} A_{\lambda} g(\lambda t).$$

We get an analogous result to Cantor-Lebesgue Lemma. It means that, if  $\dot{S}(f,t)$  converges, then the coefficients  $A_{\lambda}$  tend to zero, as  $\lambda$  goes to infinite. We extend this result for a numerate family of bounded variation functions.

### Envelope solutions for PDEs depending on two disjoint sets of variables

#### Maria Lewtchuk Espindola

Universidade Federal da Paraíba, Brazil

There are a lot of applications for the envelope solutions to PDEs, the hypersurfaces that enclose one of the families of the hypersurfaces given by the complete solutions. The development and discussion of the existence of envelope solutions to PDEs that depends of two disjoint sets of variables are the main purpose of this research. As an example it is considered the canonical variables describing a mechanical system at the phase space in Hamiltonian Analytical Mechanics. As one of the possible extensions it will be discussed the development and the analysis of the existence of envelope solutions to the variational PDEs that involves functional depending of two disjoint sets of variables. As it occurs in Hamiltonian Analytical Mechanics applied to field theories where the dependence is of the field functions and the canonical variables represented by the density momenta.

#### Stability results for impulsive functional differential equations with infinite delay and applications

#### Marta Gadotti

Universidade Estadual Paulista Júlio de Mesquita Filho, Brazil

For a family of differential equations with infinitive delay and impulses, we establish conditions for the existence of global solutions and for the global asymptotic and global exponential stabilities of an equilibrium point. The results are used to give stability criteria for a very broad family of impulsive neural network models with both unbounded distributed delays and bounded time-varying discrete delays.

#### On differentiable extensions of vector-valued functions

#### Martin Koc

RSJ a.s, Czech Republic

We briefly present an extension theorem for vector-valued Baire one functions and then use it to obtain new results on extending vector-valued functions that are differentiable at some points in a way that preserves differentiability. Results that will be presented in the talk were obtained in cooperation with Jan Kolář.

#### C-removeable sets

#### Martin Rmoutil

Charles University in Prague, Czech Republic

A subset A of a normed linear space X is C-removeable if any Lipschitz function  $f: X \to \mathbb{R}$ convex on all line segments disjoint from A is necessarily convex on X. I wish to present several new results concerning C-removeable sets in  $\mathbb{R}^n$  and related topics. A sufficient condition for Cremoveability and a counterexample will be given. This is joint work with Dušan Pokorný.

#### Generalized carachteristic equations for non-autonomous functional differential equations

#### **Miguel Vinicius Santini Frasson**

Universidade de São Paulo, Brazil

We present a result on generalized carachteristic equations, an extension of the concept of carachteristic equations for autonomous differential equations, but for functional differential equations. We give a result on the asymptotic behavior of solutions.

#### Maximal lineability of the set of continuous surjections

#### Nacib André Gurgel e Albuquerque

Universidade Federal da Paraíba, Brazil

Let m, n be positive integers. In this poster we prove that the set of all continuous and surjective functions from  $\mathbb{R}^m$  to  $\mathbb{R}^n$  contains (excluding the 0 function) a c-dimensional vector space. This result is optimal in terms of dimension.

### Reconstruction of coefficients and sources in elliptic systems modelled with many boundary

#### Nilson Costa Roberty

Universidade Federal do Rio de Janeiro, Brazil

Inverse problem for determination of unknown parameters related to both intensities and support of sources and materials coefficients in second-order elliptic equations models is posed with over specification of data on the boundary. A discrepancy function based on difference of two mixed problems formulated with a Lipschitz dissection of Cauchy data is introduced. This function controls the measured difference between the two solutions for the same set of Cauchy data. Parameters can be determined by minimization of this function under guess values. The concept of Calderón projector gap is introduced as a tool for checking the consistency of Cauchy data.

#### On the uniqueness of topological degree in infinite dimension

Pierluigi Benevieri

Universidade de São Paulo, Brazil

In some previous papers, with M. Furi, we presented a fairly simple construction of a topological degree for compact perturbations of Csp1 Fredholm maps of index zero between Banach spaces, which verifies the three fundamental properties of the classical degree theory: normalization, additivity and homotopy invariance. We show here that this degree is unique. Precisely, by an axiomatic approach similar to the one due to Amann-Weiss, we prove that there exists at most one real function satisfying the above properties, and this function must be integer valued.

### On some generalization of the classical concept of bounded variation with applications

#### Piotr Kasprzak

Adam Mickiewicz University, Poland

One of the important generalizations of the concept of the bounded variation in the sense of Jordan is the so-called  $\Lambda$ -variation, which was introduced by Waterman in 1972 in connection with his studies on the convergence of Fourier series. It can be proved that a Fourier series of a function f, which belongs to a certain class of functions of  $\Lambda$ -bounded variation, namely, to the class of functions of harmonic bounded variation, converges pointwise to the arithmetic mean of the left and right-hand side limits of f and converges uniformly to f on closed intervals of continuity of the function in question. Furthermore, each larger class contains a continuous function whose Fourier series diverges at a point. During the talk, we are going to discuss some properties of functions of  $\Lambda$ -bounded variation and present their new applications to operator theory and nonlinear differential and integral equations. In particular, we will provide sufficient conditions which guarantee that a convolution operator or a nonautonomous superposition operator maps the space of functions of  $\Lambda$ bounded variation into itself. In the second part of the talk, we are going to introduce and discuss the concept of the  $\Lambda$ -variation in the  $L^1$ -setting, which allows to deal with the  $\Lambda$ -variation of functions that are almost everywhere equal. (It is worth mentioning that the pointwise  $\Lambda$ -variation, and in particular Jordan variation, are very sensitive to modifications of the values of the given function even on a very small set.) In the case of the  $\Lambda$ -variation in the  $L^1$ -setting illustrative examples as well as applications will be also provided.

#### On the approximate variational measure

Piotr Sworowski

Casimirus the Great University, Bydgoszcz, Poland

In the talk we will discuss the connection between variational measure and generalized Riemann (Kurzweil-Henstock) integral, both with respect to some basis, emphasising on the approximate Kurzweil-Henstock integral.

#### Lower bounds on blow up solutions of the three-dimensional Navier–Stokes equations in homogeneous Sobolev spaces Ricardo Parreira da Silva

Universidade Estadual Paulista Júlio de Mesquita Filho, Brazil

Suppose that u(t) is a (weak) solution of the three-dimensional Navier–Stokes equations, either on the whole space or with periodic boundary conditions, that has a singularity at time T. We show that the norm of u(T-t) in the homogeneous Sobolev space  $\dot{H}^s$  must be bounded below by  $c_s t^{-(2s-1)/4}$  for 1/2 < s < 5/2 ( $s \neq 3/2$ ), where  $c_s$  is an absolute constant depending only on s; and by  $c_s ||u_0||_{L^2}^{(5-2s)/5} t^{-2s/5}$  for s > 5/2. (The result for 1/2 < s < 3/2 follows from well known lower bounds on blowup in  $L^p$  spaces.) We show in particular that the local existence time in  $\dot{H}^s(I\!R^3)$ depends only on the  $\dot{H}^s$ -norm for 1/2 < s < 5/2,  $s \neq 3/2$ .

Work in collaboration with: James Cooper Robinson (University of Warwick) and Witold Sadowski (Warsaw University).

#### Comparison of some trigonometric integrals

#### Tatiana Sworowska

Moscow State University, Russia

There are several integrals which recover coefficients of everywhere convergent trigonometric series from its sum by generalized Fourier formulae. Among them are Burkill's SCP-integral (Burkill J.C., Proc. London Math. Soc., 1951), James'  $P^2$ -integral (James R.D., Canad. J. Math., 1950), and Preiss–Thomson's AS-integral (Preiss D., Canad. J. Math., 1989).

We show that AS-integral is noncompatible with both SCP- and  $P^2$ -integrals, whereas it is known that the latter two are compatible.

#### Lebesgue measure on Banach space

Tepper L. Gill

Howard University, Washington DC, USA

We first develop a regular  $\sigma$ -finite version of Lebesgue measure on  $\mathbb{R}^{\infty}$ , which generates a restriction to every separable Banach space with a Schauder basis. We show that the maximal translation invariant subspace is  $\ell_1$  and the maximal rotation invariant subspace is  $\ell_2$ . I will also show the Pontryagin duality theory can be extended to all uniformly convex separable Banach spaces with a basis. In another direction, I will provide an interesting approach to the diffusion equation in infinitely many variables, which also works for the Ornstein-Uhlenbech equation.

#### Gagliardo-Nirenberg estimates for localizable Hardy-Sobolev spaces

#### **Tiago Henrique Picon**

Universidade Federal de São Carlos, Brazil

In this lecture we will discuss the validity of the Gagliardo-Nirenberg estimates for homogeneous localizable Hardy-Sobolev spaces  $\dot{h}^{1,p}(I\!\!R^N)$  when 0 .

#### On very non-linear subsets of continuous functions

#### Vinícius Vieira Fávaro

Universidade Federal de Uberlândia, Brazil

In this work we continue the study initiated by Gurariy and Quarta in 2004 on the existence of linear spaces formed, up to the null vector, by continuous functions that attain the maximum only at one point. Inserting a topological flavor to the subject, we prove that results already known for functions defined on certain subsets of  $\mathbb{R}$  are actually true for functions on quite general topological spaces. In the line of the original results of Gurariy and Quarta, we prove that, depending on the desired dimension, such subspaces may exist or not.

More precisely, given a topological space D, by  $\widehat{C}(D)$  we denote the subset of the linear space C(D) of all real-valued continuous functions on D composed by the functions that attain the maximum exactly once in D.

The main results obtained by Gurariy and Quarta in this direction are the following:

- (A)  $\widehat{C}[a,b)$  contains, up to the origin, a 2-dimensional linear subspace of C[a,b).
- (B)  $\widehat{C}(\mathbb{R})$  contains, up to the origin, a 2-dimensional linear subspace of  $C(\mathbb{R})$ .
- (C) There is no 2-dimensional linear subspace of C[a, b] contained in  $\widehat{C}[a, b] \cup 0$ .

In this work we extend (A) to spaces of functions defined on topological spaces D that can be continuously embedded onto some Euclidean sphere  $S^n$ . We also extend (B) to spaces of functions defined on quite general topological spaces D that include  $\mathbb{R}$ . In the two former cases we prove that  $\widehat{C}(D) \cup 0$  contains an (n + 1)-dimensional subspace. Finally, we extend (C) to spaces of functions defined on compact subsets K of  $\mathbb{R}^m$ . In this case we prove that  $\widehat{C}(K) \cup 0$  does not contain an (m + 1)-dimensional subspace of C(K) for every compact  $K \subset \mathbb{R}^m$  but, on the other hand, there are compact sets  $K \subset \mathbb{R}^m$  for which  $\widehat{C}(K) \cup 0$  contains an m-dimensional subspace of C(K).

This is a joint work with G. Botelho, D. Cariello, D. Pellegrino and J.B. Seoane-Sepúlveda.

Programme

	MONDAY 3	TUESDAY 4	WEDNESDAY 5	THURSDAY 6
Chairman	Tepper L. Gill	Jun Kawabe	Dariusz Bugajewski	Pierluigi Benevieri
8:20 - 9:00	OPENING SECTION	Emanuel Carneiro	Paul Humke	Daniel Pellegrino
9:00 - 9:40	Jean Mawhin	Dimitar Dimitrov	Milan Tvrdy	Patrick Muldowney
9:40 - 10:20	Artur Lopes	Krzysztof C. Ciesielski	Richard Aron	Jan Maly
10:20 - 10:50	COFFEE BREAK	COFFEE BREAK	COFFEE BREAK	COFFEE BREAK
Chairman	Nilson C. Roberty	Marcone C. Pereira	Marcos Napoleão	Piotr Sworowski
10:50 - 11:15	Jun Kawabe	Tepper L. Gill	Pierluigi Benevieri	Eduardo S. G. Leandro
11:15 - 11:40	Iguer L. D. Santos	Martin Rmoutil	Eliza R. Santos	Dariusz Bugajewski
11:40 - 12:05	Ricardo P. da Silva	Piotr Sworowski	Marcone C. Pereira	Igor Mencattini
12:05 - 14:00	LUNCH	LUNCH		LUNCH
Chairman	Martin Rmoutil	Eduard Toon		Piotr Kasprzak
14:00 - 14:25	Eduard Toon	Antonio R. da Silva		Nacib G. Albuquerque
14:25 - 14:50	Piotr Kasprzak	Maria L. Espindola		María G. M. Macías
14:50 - 15:15	Jaqueline G. Mesquita	Tatiana Sworowska		Francisco J. M. Torres
15:15 - 15:45	COFFEE BREAK	COFFEE BREAK	BARBECUE	CLOSING SECTION
Chairman	Tiago H. Picon	Martin Koc		
15:45 - 16:10	Nilson C. Roberty	Luciano Barbanti		
16:10 - 16:35	Martin Koc	Marta Gadotti		
16:35 - 17:00	Inajara da S. Freitas	Vinícius V. Fávaro		
17:00 - 17:25	Laurent Moonens	Tiago H. Picon		
17:25 - 17:50	Marcos Napoleão	Miguel Frasson		
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